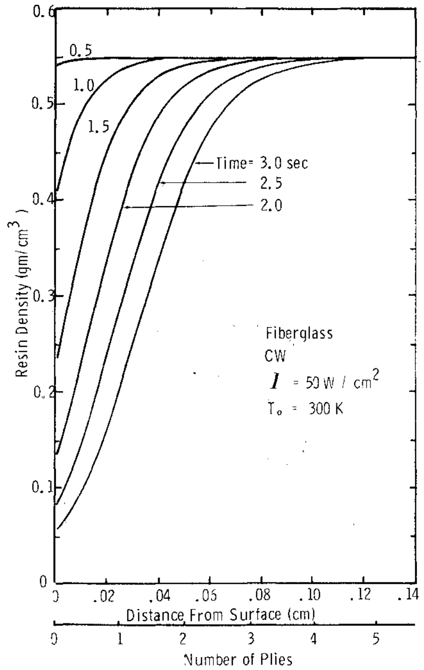

Fig. 4 Resin density distributions at various times for an equivalent cw laser.

As shown in Fig. 2, a residual energy of 5 J/cm^2 can be deposited in fiberglass with pulse fluence of $F \simeq 8$ J/cm^2 (without resin) and $F \simeq 20$ J/cm^2 (with resin). Let's consider a 10-pps pulsed laser which delivers an average residual energy of 5 J/cm^2 per pulse. The average intensity for an equivalent CW laser is 50 W/cm^2. With an average intensity of 50 W/cm^2, the one-dimensional heat conduction analysis provides the temperature and density history. As shown in Fig. 4, the resin density profiles are given at various times. At the time of about 2.5 s, the resin content at the interface between the first two plies has been reduced by a factor of 2. Hence, if this criterion is used for ply removal, it is possible to remove the first ply on a fiberglass surface with a 10-pps laser which delivers an average residual energy of 5 J/cm^2 per pulse in approximately 2.5. s.

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Proportional Optimal Orthogonalization of Measured Modes

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Introduction

METHODS for orthogonalization of measured mode shapes have been proposed in the literature by several authors.[1-9] In Refs. 9-11, the orthogonalized modes and their measured frequencies were used to correct a given stiffness matrix. In Ref. 10, the present author proposed a method by which the rigid body modes are not corrupted, and the measured credibility of the different groups of measured modes is incorporated by the order of their selection during the orthogonalization process. In fact, Ref. 10 tries to satisfy the seemingly opposed opinions of Rodden[12] and Targoff.[13] Rodden[12] requires the orthogonalization method to keep the rigid body modes uncorrupted and to assign a higher credibility to the measurements of lower-frequency modes, while Targoff[5] thinks that modes which occur in grouping with narrow frequency band must be equally treated. However, the selective method proposed in Ref. 10 satisfies the first or second requirements in a discrete way which cannot be easily controlled. By the method presently proposed,[14] the requirements can be satisfied smoothly and in a controllable way. The different requirements can be satisfied in a simple way by introducing a properly chosen matrix of proportionality. In this way, the corrupted mode shapes are obtained simultaneously. In the numerical example, given "measured" modes are orthogonalized by applying the two methods. The results show clearly the advantages of the presently proposed method. It is interesting to show the relationship among the proposed methods and those of McGrew[3] and Targoff.[8] The selective method degenerates to the McGrew method when the shape modes are selected for orthogonalization one by one. If one chooses the matrix of proportionality to be the unit matrix and orthogonalizes simultaneously all mode shapes, including the rigid body modes, the proportional method[14] degenerates to that of Targoff.[13]

Formulation of the Problem and Its Solution

Following the modified method[10] of the basic approach given in Ref. 9, one must first select the rigid body mode shapes. This must be done in order to keep these modes uncorrupted. Let $R(n \times r)$ be a matrix which represents the analytically known rigid body mode shapes which have already been orthogonalized. Hence,

$$R^t M R = I \tag{1}$$

where $M(n \times n)$ is a known symmetric positive definite mass matrix.

Let $T(n \times q)$ be a matrix which represents the modes which have to be orthogonalized. It must be noted[9,10] that the measured modes T_i have to be normalized in the following way

$$T_i = \tilde{T}_i (\tilde{T}_i^t M \tilde{T}_i)^{-1/2} \tag{2}$$

where $\tilde{T}_i$ is the mode shape before normalization.

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We are looking now for a matrix $Q(n \times q)$ which satisfies the weighted orthogonality conditions

$$Q^t M Q = I \tag{3}$$

and which minimizes a given norm. We propose here to minimize the norm

$$\psi = \| N(Q-T)\alpha \| = n_{ij}(q_{jk} - t_{jk})\alpha_{k\ell} n_{ip}(q_{pq} - t_{pq})\alpha_{q\ell} \tag{4}$$

where the Einstein rule of summation is applied. $N(n \times n)$ is the positive definite symmetric solution of the relation

$$N = M^{1/2} \tag{5}$$

$$\alpha = \lceil \alpha \rfloor \tag{6}$$

where $\alpha(q \times q)$ is a diagonal matrix of proportionality, or a matrix of credibility. Clearly, a higher value of α_i will be assigned to a mode shape T_i with higher measurement credibility. If one assumes that the measured modes have their errors apportioned in accordance[12] to their modal frequencies, then α_i must be proportional to the reciprocal value of the suitable measured frequency. On the other hand, if someone wants a grouping of modes with narrow frequency band to be equally treated,[13] he must assign the same credibility value α_i to any one of the modes belonging to this group. If α is taken to be the unit matrix and T represents all measured modes including the rigid body modes, the case degenerates to that treated in Ref. 9.

Only experiments and experience will teach us what the proper values are of the credibility matrix α. For now, we must leave the assignment of these values to the discretion and intuition of the practicing engineer. However, we will assume here that the credibility matrix α is known.

Following Refs. 9 and 10, the problem can now be cast in the following mathematical form: Given are the orthogonalized rigid body modes R, Eq. (1); the measured modes T, Eq. (2); the positive definite mass matrix M, Eq. (1); and the credibility matrix α, Eq. (6); find a matrix Q which minimizes the norm (4) and satisfies the constraint (3) and an additional constraint (7)

$$R^t M Q = 0 \tag{7}$$

Constraint (7) requires that matrix Q be orthogonal to the already known rigid body modes.

The constraints (3) and (7) can be incorporated into the cost function (4) by using Lagrange multipliers. In this way, the following Lagrange function is obtained

$$G = \psi + \lambda_{i\ell}(q_{ji} m_{jk} q_{k\ell} - \delta_{i\ell}) + 2\beta_{is} q_{ji} m_{jp} r_{ps} \tag{8}$$

where $\delta_{i\ell}$ is the Kronecker delta, β is a matrix of order $(q \times r)$, and Λ is a matrix of order $(q \times q)$. The symmetry of Eq. (3) requires Λ to be symmetric

$$\Lambda^t = \Lambda \tag{9}$$

The partial differentiation of Eq. (8) in respect to q_{fg}, where the results are equated to zero, yields equations which q_{fg} have to satisfy when G is minimal

$$\frac{\partial G}{\partial q_{fg}} = 2n_{if}\alpha_{g\ell} n_{ip}(q_{pq} - t_{pq})\alpha_{q\ell} + 2\lambda_{g\ell} m_{fk} q_{k\ell} + 2\beta_{gs} m_{fp} r_{ps} = 0 \tag{10}$$

Written in matrix form, Eq. (10) becomes

$$\frac{\partial G}{\partial Q} = 2M(Q-T)\alpha^2 + 2MQ\Lambda + 2MR\beta^t = 0 \tag{11}$$

The matrix M is invertible and Eq. (11) yields

$$Q(\alpha^2 + \Lambda) - T\alpha^2 + R\beta^t = 0 \tag{12}$$

Multiplying Eq. (12) by $R^t M$ and taking into account Eqs. (1) and (7) yields

$$\beta^t = R^t M T \alpha^2 \tag{13}$$

By substitution of Eq. (13) into Eq. (12), one obtains

$$Q(\alpha^2 + \Lambda) = P \tag{14}$$

where $P(n \times q)$ is given by

$$P = T\alpha^2 - RR^t MT\alpha^2 = (I - RR^t M)T\alpha^2 \tag{15}$$

Assuming that $\alpha^2 + \Lambda$ is invertible and substituting Eq. (14) into Eq. (3) yields

$$(\alpha^2 + \Lambda) = (P^t MP)^{1/2} \tag{16}$$

and by substitution of Eq. (16) into Eq. (14), one finally obtains

$$Q = P(P^t MP)^{-1/2} \tag{17}$$

It can be shown[9] that Eq. (17) yields the minimum of ψ provided that the positive root of $P^t MP$ is used in Eq. (17).

Several techniques for the solution of Eq. (17) are described in Refs. 9, 15-19. Since the introduction of the credibility matrix α causes some changes in the behavior of the matrices appearing in Eq. (17), these techniques have to be properly modified. For more details, see Ref. 14.

It will be assumed for convenience that $\alpha_i > 0$. In addition, it must be noted that the relationship between the different values of α_i is important and not their absolute value. Hence, the matrix α can be multiplied by any nonzero scalar which,

Table 1 Orthogonalization of the modes given in Eq. (20)

α_1	α_2	α_3	α_4	α_5	α_6	e_1	e_2	e_3	e_4	e_5	e_6
1) Simultaneous proportional orthogonalization of all modes											
1.000	1.000	1.000	1.000	1.000	1.000	0.0243	0.0243	0.0243	0.0243	0.0243	0.0243
1.000	0.851	0.721	0.610	0.513	0.430	0.0095	0.0157	0.0231	0.0311	0.0393	0.0474
1.000	0.524	0.270	0.138	0.0069	0.0034	0.0013	0.0161	0.0336	0.0491	0.0617	0.0719
2) Selection of the first mode; simultaneous orthogonalization of the others											
∞	1.000	1.000	1.000	1.000	1.000	0.0000	0.0401	0.0401	0.0401	0.0401	0.0401
∞	1.000	0.851	0.721	0.610	0.513	0.0000	0.0314	0.0360	0.0413	0.0471	0.0531
∞	1.000	0.275	0.0073	0.0019	0.0005	0.0000	0.0248	0.0405	0.0549	0.0665	0.0758
3) One by one selective orthogonalization											
						0.000	0.0247	0.0428	0.0569	0.0679	0.0769

for convenience again, will be taken positive.

Let the matrix $X(n\times n)$ be the modal matrix which represents all already corrected mode shapes, so that

$$X^tMX=I \tag{18}$$

Following Refs. 9 and 11, X can be used to obtain an optimally corrected stiffness matrix $Y(n\times n)$ from a given stiffness matrix $K(n\times n)$ [see Ref. 9, Eq. (28) or Ref. 11, Eq. 23)].

$$Y=K-KXX^tM-MXX^tK+MXX^tKXX^tM+MX\Omega^2X^tM \tag{19}$$

where $\Omega^2\,(m\times m)$ represents the measured frequencies, which for the rigid body modes are zero. Note that Y incorporates the measured frequencies and their orthogonalized modes. Equation (19) can be used for further dynamic calculations.

Numerical Examples

To test the method, a "measured" modal matrix $\tilde{T}(11\times 6)$ was chosen so that the nonorthogonality between any two modes was the same $(T_i^tMT_j=0.157,\ i\neq j)$. The mass matrix was chosen to be the unit matrix I.

$$\tilde{T}=\begin{bmatrix} 1 & 1 & & & & \\ 1 & & 1 & & & \\ 1 & & & 1 & & \\ 1 & & & & 1 & \\ 1 & & & & & 1 \\ 1 & & & & & \\ 1.061 & & & & & \\ 1 & & & & & \\ 1 & & & & & \\ 1 & & & & & \\ 1 & -0.431 & -0.431 & -0.431 & -0.431 & -0.431 \end{bmatrix} \tag{20}$$

The main interest of the test was to find the influence of the credibility matrix α upon the change between the given measured mode T_i and its orthogonalized counterpart Q_i. Following Eq. (3), a measure of this change can be defined as follows:

$$e_i=(Q_i-T_i)^tM(Q_i-T_i) \tag{21}$$

Since M is a positive definite symmetric matrix e_i would be zero only for T_i equal to Q_i. The iterative technique given in Eq. (24) of Ref. 14 was applied to orthogonalize the given modes. Between 6 and 41 iterations were needed to obtain a nonorthogonality ($|Q_i^tMQ_j|$, $i\neq j$) less than 10^{-8}.

In the first test, all the modes given in Eq. (20) were orthogonalized simultaneously. The credibility matrix α was changed gradually from a unit matrix to a matrix with steeply descending values of α_i. In the second test, the first mode was selected first as the "known mode," and the other modes were simultaneously orthogonalized using again a gradually changing matrix α. Finally, the modes were orthogonalized one by one using the selective technique given in Ref. 10. Some typical results are given in Table 1.

As expected, the changes e_i, between the measured modes, T_i, and the corrected modes Q_i ascend for decending values of their credibility values α_i. It is interesting to see that the results obtained by the selective technique can be achieved by the technique proposed here, simply by using a properly chosen credibility matrix α. A comparison between the last line of test 2 and test 3 emphasizes this important fact. However, as stated before, while the selective technique is uncontrollable, the credibility matrix α permits us to have some quantitative control upon the error numbers e_i. Clearly, the first mode shape in tests 2 and 3 is uncorrupted.

Conclusions

A method has been described in which measured modes are orthogonalized by using a matrix of proportionality. In this way, the different measurement credibilities of the measured modes can be incorporated in a quantitative way, and the rigid body modes can be selected and kept uncorrupted.

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